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ANALYSIS OF TIME-HISTORY DATA OF FORCES AND MOTIONS MEASURED AT TOWING FACILITIES

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NOTATION

A	Amplitude of time-history data
h_n	Hanning window function
S	Power spectral density
Δt	Sampling time interval
T_t	Total time
ω	Frequency
ϕ	Phase angle

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ABSTRACT

Methods to filter and to analyze measured data in towing tanks are described and the numerical results are validated with the generated data. A lowpass filter is used to eliminate the noise of the measured data. The methods of zero crossing, sine function, and spectral analysis are applied to compute the amplitudes, periods, and phase angles. The numerical results of the measured data are compared with those of the different methods. When the measured data is near sinusoidal, the results analyzed with the three methods agree very well. The computer program, HARMON is written to perform this analysis.

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INTRODUCTION

When sinusoidal water waves are generated by the wave makers in towing facilities, the measured forces or motion data of the body being tested in the facility are expected to be generally sinusoidal. Due to the physics of real waves, in addition to the noise in measurement equipment, the recorded data are rarely sinusoidal. A numerical method to analyze the recorded time-history data is necessary to find the amplitudes and phase angles accurately. The present study will discuss the filtering of recorded data with noise and the analysis method. Three approaches will be discussed: the zero-crossing method, the method of sine function and spectral analysis.

When we analyze many data sets at one time, we must have a quick way to analyze these data and to verify them. The main advantage of having three methods is that, by comparing the analyzed data with these methods, we can distinguish good data from bad data easily. When the recorded data are nearly sinusoidal, the analyzed results from these methods are nearly identical within the limit of numerical accuracy.

Generated data are used to demonstrate the validation of each of these methods. The numerical results are presented in this report for the analyses of the forces, moments, and motions of a measured data set.

DATA PROCESSING METHODS

(A) FILTERING

Recorded time-history data usually contains noise due to measurement equipment or vibration of the towing carriage. The noise in the towing tanks is usually of high frequency and a lowpass filter can be used to remove it. The relationship of the raw and filtered data is as follows:

$$Y(t) \quad = \quad \int_{-\infty}^{\infty} h(\tau) X(t-\tau) d\tau \tag{1}$$

The function, h(t) is a filter since the time-history raw data X(t) is converted into the time-history output Y(t). The Fourier transform of h(t) becomes the transfer function, H(jω), which is a periodic function of ω , equal to $2\pi/\Delta t$, as described in Reference 1. Δt is the sampling interval. The filter function is expressed as the complex Fourier coefficient formula for the analysis of the finite data

$$h_n \quad = \quad \frac{\Delta t}{2\pi} \int_{-\pi/\Delta t}^{\pi/\Delta t} H(j\omega) e^{jn\omega\Delta t} d\omega, \qquad -N \leq n \leq N \tag{2}$$

where (2N+1) is defined as the number of samples in the moving window.

When H(jω) is real, Equation (1) simplifies to

$$h_n \quad = \quad \frac{\Delta t}{\pi} \int_{0}^{\pi/\Delta t} H(j\omega) \cos(n\omega\Delta t) d\omega \quad = \quad h_{-n} \tag{3}$$

In Equations (2) and (3), j denotes square root of negative one.

A lowpass non-recursive digital filter with no phase shift at any frequency is a filter used to eliminate high-frequency noise while passing a low-frequency signal without changing its phase angle. The desired transfer function, $H_d(j\omega)$ is shown in Figure 1 in periodic form. The gain is equal to one for frequencies smaller than or equal to the cutoff frequency, ω_c and is zero elsewhere.

Equation (3) now gives the filter coefficients

$$h_n \quad = \quad \frac{\Delta t}{\pi} [\int_{0}^{\omega_c} 1 \cdot \cos(n\omega\Delta t) d\omega + \int_{\omega_c}^{\pi/\Delta t} 0 \cdot \cos(n\omega\Delta t) d\omega] \quad = \quad \frac{\omega_c \Delta t}{\pi} \frac{\sin(n\omega_c \Delta t)}{n\omega_c \Delta t} \tag{4}$$

When n becomes zero, Equation (4) is expressed as

$$h_o = \frac{\omega_c \Delta t}{\pi} \tag{5}$$

Since $H_d(j\omega)$ in Figure 1 causes sharp discontinuities in the function, h_n, the Hanning window is applied to Equation (4) as follows:

$$x(t) = \frac{1}{2}[1 + \cos(\frac{t\pi}{N\Delta t})] \tag{6}$$

The final lowpass filter is $h_n x(t)$ and is expressed as follows:

$$H_l(n,t) = \frac{1}{2n\pi}\sin(n\omega_c \Delta t)[1 + \cos(\frac{t\pi}{N\Delta t})] \tag{7}$$

The Hanning window and lowpass filtering functions with and without the Hanning function are plotted in Figure 2.

The desired transfer function for a highpass non-recursive digital filter with no phase shift is given in Figure 1b and Equation (3) now gives the filter coefficients

$$\begin{aligned} h_n &= \frac{\Delta t}{\pi}[\int_0^{\omega_c} 0 \cdot \cos(n\omega\Delta t)d\omega + \int_{\omega_c}^{\pi/\Delta t} 1 \cdot \cos(n\omega\Delta t)d\omega] \\ &= \frac{\sin(n\pi)}{n\pi} - \frac{\sin(n\omega_c \Delta t)}{n\pi} \end{aligned} \tag{8}$$

Similar to the case of the lowpass filter, the digital process for the highpass filter is multiplied by the Hanning window, which is Equation (6). The details of the filtering process are given in Reference 1.

To verify the filtering process numerically, the following mathematical function is used, which is representative of a sinusoidal signal with a period of 3.27 seconds, with some noise introduced. This signal is plotted in Figure 3a.

$$y(t) = A_1 \sin(\omega_1 t + \phi_1) + A_2 \sin(\omega_2 t + \phi_2) \tag{9}$$

where

$$A_1 = 5.0, \quad \omega_1 = 1.92, \quad \phi_1 = \frac{\pi}{6}, \quad A_2 = 1.0, \quad \omega_2 = 9.42, \quad \phi_2 = \frac{\pi}{4} \tag{10}$$

The cutoff frequency is found as 1.69 times the lower frequency by a trial and error method. When the cutoff frequency is 1.69 times the lower frequency, the filtered data are close to the exact values. The value of N is calculated as follows:

$$N = \frac{4\pi}{\omega_c \Delta t} + 1 \tag{11}$$

If a time step of $\Delta t = 0.01$ is used, N becomes 388. The Hanning window and lowpass filtering functions with and without the Hanning function are plotted in Figure 2. The role of the Hanning window is to reduce the magnitude of the filtering function, Equation (4), at the ends of filtering time period.

The raw and filtered data are shown in Figure 3. If the filtering method is effective, the filtered data will match the first term of Equation (9). The filtered data without the Hanning window (Fig. 3b) show slightly smaller amplitudes than the exact amplitudes, which are generated by the first term of Equation (9). The plots of Figure 3b are magnified in Figure 4. Figure 4 shows that the values between the upper and lower peaks seem the same, but the midpoint of each half cycle oscillates, contrary to that of the filtered data with the Hanning window in Figure 3c. The agreement between the amplitudes of the filtered data with the Hanning window and the exact data is very good.

To filter the data using a highpass filter, Equation (8) and the Hanning window are used, which are shown in Figure 5. The highpass signal could also be obtained by subtracting the lowpass signal from the original data. The filtered data with and without the Hanning window are shown in Figure 6. When the time is less than 3.94 seconds, which is the filtering interval, the filtered data are not usable since the raw data are assumed to be zero for time prior to zero seconds. This assumption means that the filtered data for time less than 3.94 seconds or for time greater than the total time minus 3.94 seconds are not filtered correctly. The exact data in Figure 6 are the "noise" data from the second term of Equation (9). The amplitudes of the filtered data without the Hanning window fluctuate in time as with the lowpass filter. The filtered data with the Hanning window agree with the exact data as shown in Figure 6.

(B) SPECTRAL ANALYSIS

When the measured data contains periodic signals, the amplitude and period of these signals can be computed through power spectral analysis via the Fourier transform of the time-history data. If the measured data are expressed as y(t), the Fourier transform will be

$$Y(\omega) = \int_{-\infty}^{\infty} y(t) e^{i\omega t} dt \tag{12}$$

The inverse transform of Equations (12) is

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(\omega)e^{-i\omega t}\,d\omega \tag{13}$$

If T_t is the total time for measuring the data, the power spectral density is defined as (Reference 2)

$$S(\omega) = \lim_{T_t \to \infty} \frac{1}{T_t}\left|\int_{-T_t/2}^{T_t/2} y(t)e^{i\omega t}\,dt\right|^2 \tag{14}$$

For the analysis of the measured data, the fast Fourier Transform (FFT) is used instead of Equation (12) or (13) for faster computing as detailed in Reference 3. Among the various methods of FFT listed in Reference 3, the subroutine REALFT is used for the present study. The results of REALFT of y(t) will be complex values as a function of the frequency, ω. The square of the magnitude of the complex values becomes the spectral density and the amplitude of the measured data will be the square root of two times the area of the spectral density curve, shown in Equation (14). The FFT of a data set is symmetric. REALFT uses only the positive range, so the results of REALFT will be half of the total of Equation (14).

The power spectral density of a sinusoid with an amplitude of one ($y(t)=\sin\omega_o t$ with $\omega_o=2.45$) is plotted in Figure 7, which is a delta function. Since the base of the area is two times the frequency step, the area under the curve is two times the total power and the square root of this area will be the amplitude of the time-history data. The simple method to compute the area under the curve is that it is assumed to be a triangle. It is found that the area of ΔABC is 1.57. The square root of this value, which is the amplitude of the sine function, is 1.25. The original amplitude of the sine function is one. This indicates that the assumption of triangular area under the spectral density curve is not correct. In order to find the accurate amplitude of the time-history data, the section AB in Figure 7c is assumed as a parabola, which is a convenient choice, as

$$f(\omega) = a(\omega - \omega_1)^2 + f_1 \tag{15}$$

where

$$a = \frac{f_2 - f_1}{(\omega_2 - \omega_1)^2} \tag{16}$$

The area under the parabola is the integral of Equation (15) with the integral range between ω_1 and ω_2 as

$$Area = \frac{1}{3}(\omega_2 - \omega_1)(f_1 + f_2) \tag{17}$$

With the substitution of the values in Figure 7 into Equation (17), the area under the parabola is 0.524, which is half of the total area. The computed amplitude is 1.02.

(C) ZERO-CROSSING METHOD

The zero-crossing method can be used to find the amplitude and period of sinusoidal or quasi-sinusoidal data. This method involves finding the zero crossing of the data, and computing the area of the positive and negative parts. The area can then be used to calculate the amplitude of the sine curve. The area of the half cycle of a sine curve with an amplitude of a and a frequency of ω is as follows:

$$\int_0^{\pi/\omega} a\sin(\omega t)dt = \frac{2a}{\omega} = A \tag{18}$$

where a is an amplitude and A is the area under the half cycle of a sine curve. The half period will be the time interval between two consecutive times where the magnitudes are up or down crossing zero. By averaging the times between successive zero crossings, we can obtain an average half period of the filtered data.

To find the amplitude of the filtered data, the areas of the positive and negative magnitudes for half cycles as shown in Figure 8 are computed and the average of the absolute values of these areas will be the area of half cycle of a sinusoidal curve. Then, the amplitude will be computed by equating the average area with Equation (18) as follows:

$$a = \frac{\omega(A_1 + |A_2|)}{4} \tag{19}$$

The present method will be numerically validated with the generated data. For the single wave, the results of the first term of Equation (9) are used as the generated data for the validation. The computed amplitude is 5 as given in Table 1. In Table 1, the results of other methods, which will be described later, are also included.

(D) METHOD OF SINE FUNCTION

(i) SINGLE-FREQUENCY WAVE

When the frequency of the filtered data is known, a new method to find the amplitudes and phase angles of the measured data will be given here for the single-frequency data, referred to as the "method of sine function". Let the measured data be expressed for a single wave as follows:

$$y = A\sin(\omega t + \phi) \tag{20}$$

where A is the amplitude, ω the frequency, t is time, and ϕ is the phase angle. Equation (20) can be expanded as follows:

$$y = A\sin\omega t\cos\phi + A\cos\omega t\sin\phi = A_c\sin\omega t + A_s\cos\omega t \tag{21}$$

where

$$A_c = A\cos\phi;\ \ A_s = A\sin\phi;\ \ A = \sqrt{A_c^{\ 2} + A_s^{\ 2}} \tag{22}$$

and

$$\phi = \tan^{-1}\left(\frac{A_s}{A_c}\right) \tag{23}$$

To compute the amplitudes, A_c and A_s in Equation (21), we substitute two measured data points at the times of t_1 and t_2 into Equation (21).

$$y(t_1) = A_c\sin\omega t_1 + A_s\cos\omega t_1 \tag{24}$$

$$y(t_2) = A_c\sin\omega t_2 + A_s\cos\omega t_2 \tag{25}$$

The amplitudes, A_c and A_s in Equations (24) and (25) can be computed easily by solving these equations simultaneously. However, when the values of the sine and cosine functions become small, the numerical approach will cause an error from dividing by a small number. When the argument of sine and cosine functions is 45 degrees, the values of the sine and cosine functions are both maximized. To minimize error, the argument of the sine and cosine functions in Equation (21) should be as follows:

$$\omega t = \frac{\pi}{4} + m\pi,\ \ \text{m=0, 1, 2, ...} \tag{26}$$

To have the values of the sine and cosine functions in Equation (21) maximal, the measured data at the following values of time should be used

$$m = \frac{\omega t}{\pi} - \frac{1}{4} \tag{27}$$

Other values for t can also be used as long as the values of $\sin\omega t$ and $\cos\omega t$ are not small. Equation (27) is a convenient choice, but if the precise value cannot be obtained, this is not important. The values of times t_1 and t_2 in Equations (24) and (25) will be the values of t in Equation (27) for m equal to zero and integers. In general, the time in Equation (27) does not exactly match the time of the recorded data because it is dependent on the time step of that particular data set. The time step used should be the closest time step near the computed time of Equation (27). There are two solutions of Equation (21) for each cycle of the measured data and therefore, there are twice as many solutions as there are cycles of the measured data. The average of these amplitudes and phase angles will be the final values for the amplitude and phase angle.

The present method is numerically validated with the generated data. For the single wave, the results of the first term of Equation (9) are used as the generated data for the validation. The total number of data points is 9000 with a time step of 0.01 second and with the value of ω_1, in Equation (9), m is found to be 27 different values in this case from Equation (27). There are 13 sets of Equations (24) and (25). The averages of the solutions of these equations become the final amplitude and phase angle. The computed amplitude is 5.0 and the computed phase angle is 30.0 degrees as given in Table 1.

(ii) TWO-FREQUENCY WAVE

For the two-frequency wave the measured data can be expressed as follows:

$$y = A_1 \sin(\omega_1 t + \phi_1) + A_2 \sin(\omega_2 t + \phi_2) + A_3 \sin[(\omega_1 - \omega_2)t + \phi_3] \tag{28}$$

where the subscript 1 represents the values of the first frequency and the subscript 2 represents the values of the second frequency. Equation (28) will be expanded as follows:

$$\begin{aligned} y = {} & A_{1c} \sin\omega_1 t + A_{1s} \cos\omega_1 t + A_{2c} \sin\omega_2 t + A_{2s} \cos\omega_2 t \\ & + A_{3c} \sin(\omega_1 - \omega_2)t + A_{3s} \cos(\omega_1 - \omega_2)t \end{aligned} \tag{29}$$

where

$$A_1 = \sqrt{A_{1c}^{\ 2} + A_{1s}^{\ 2}} \tag{30}$$

$$A_2 = \sqrt{A_{2c}^{\ 2} + A_{2s}^{\ 2}} \tag{31}$$

$$A_3 = \sqrt{A_{3c} + A_{3s}} \tag{32}$$

$$\phi_1 = \tan^{-1}\left(\frac{A_{1s}}{A_{1c}}\right) \tag{33}$$

$$\phi_2 = \tan^{-1}\left(\frac{A_{2s}}{A_{2c}}\right) \tag{34}$$

$$\phi_3 = \tan^{-1}\left(\frac{A_{3s}}{A_{3c}}\right) \tag{35}$$

To find the amplitudes, A_{1c}, A_{1s}, A_{2c}, A_{2s}, A_{3c} and A_{3s} in Equation (29), we substitute the measured data at the times, t_1, t_2, t_3, t_4, t_5 and t_6 into Equation (29) as follows:

$$\begin{aligned} y(t_1) = {} & A_{1c}\sin\omega_1 t_1 + A_{1s}\cos\omega_1 t_1 + A_{2c}\sin\omega_2 t_1 + A_{2s}\cos\omega_2 t_1 \\ & + A_{3c}\sin(\omega_1 - \omega_2)t_1 + A_{3s}\cos(\omega_1 - \omega_2)t_1 \end{aligned} \tag{36}$$

$$\begin{aligned} y(t_2) = {} & A_{1c}\sin\omega_1 t_2 + A_{1s}\cos\omega_1 t_2 + A_{2c}\sin\omega_2 t_2 + A_{2s}\cos\omega_2 t_2 \\ & + A_{3c}\sin(\omega_1 - \omega_2)t_2 + A_{3s}\sin(\omega_1 - \omega_2)t_2 \end{aligned} \tag{37}$$

$$\begin{aligned} y(t_3) = {} & A_{1c}\sin\omega_1 t_3 + A_{1s}\cos\omega_1 t_3 + A_{2c}\sin\omega_2 t_3 + A_{2s}\cos\omega_2 t_3 \\ & + A_{3c}\sin(\omega_1 - \omega_2)t_3 + A_{3s}\sin(\omega_1 - \omega_2)t_3 \end{aligned} \tag{38}$$

$$\begin{aligned} y(t_4) = {} & A_{1c}\sin\omega_1 t_4 + A_{1s}\cos\omega_1 t_4 + A_{2c}\sin\omega_2 t_4 + A_{2s}\cos\omega_2 t_4 \\ & + A_{3c}\sin(\omega_1 - \omega_2)t_4 + A_{3s}\sin(\omega_1 - \omega_2)t_4 \end{aligned} \tag{39}$$

$$\begin{aligned} y(t_5) = {} & A_{1c}\sin\omega_1 t_5 + A_{1s}\cos\omega_1 t_5 + A_{2c}\sin\omega_2 t_5 + A_{2s}\cos\omega_2 t_5 \\ & + A_{3c}\sin(\omega_1 - \omega_2)t_5 + A_{3s}\sin(\omega_1 - \omega_2)t_5 \end{aligned} \tag{40}$$

$$\begin{aligned} y(t_6) = {} & A_{1c}\sin\omega_1 t_6 + A_{1s}\cos\omega_1 t_6 + A_{2c}\sin\omega_2 t_6 + A_{2s}\cos\omega_2 t_6 \\ & + A_{3c}\sin(\omega_1 - \omega_2)t_6 + A_{3s}\cos(\omega_1 - \omega_2)t_6 \end{aligned} \tag{41}$$

The solution for A_{1c}, A_{1s}, A_{2c}, A_{2s}, A_{3c} and A_{3s} is similar to the solution of Equations (24) and (25). To find the time for which the values of the sine and cosine functions are maximal, the arguments of the sine and cosine functions should be as follows:

$$\omega_1 t = \frac{\pi}{4} + m\pi, \quad m=0, 1, 2, \ldots\ldots \tag{42}$$

$$\omega_2 t = \frac{\pi}{4} + n\pi, \quad n=0, 1, 2, \ldots. \tag{43}$$

To compute the exact solution for the integers m and n of Equations (42) and (43) is difficult. An approximate method is applied to the solution for these two equations. First the time for the zero value of the data is found, and the half periods of two frequencies are divided by three different intervals by avoiding the small values of sine and cosine functions. There are twice as many values for m of Equation (42) as the number of Equations (36) to (41).

For the validation of two-frequency waves, the results of Equation (9) are used as the generated data. There are 21 sets of the solutions of Equations (36) and (39). Since Equation (8) does not contain the term for the difference frequencies, only four equations are used to validate the method. Therefore, there are the 5 sets of Equations (36) through (39) and the averages of the solutions of 5 sets will be the final amplitudes and phase angles. The computed amplitudes are 5.0 and 1.0, and the computed phase angles are 30 and 45 degrees as given in Table 2. In Table 2, the results of spectral analysis are also included for comparison.

Since the measured data for two-frequency forces are not available, these data are generated with the following equation

$$\begin{aligned} y(t) = {} & A_1 \sin(\omega_1 t + \phi_1) + A_2 \sin(\omega_2 t + \phi_2) \\ & + A_3 \sin[(\omega_1 - \omega_2)t + \phi_3] + A_4 \sin(\omega_3 t) \end{aligned} \tag{44}$$

where

$$\begin{aligned} & A_1 = 10.0, \quad \omega_1 = 2.827, \quad \phi_1 = 30^o \\ & A_2 = 5.0, \quad \omega_2 = 2.199, \quad \phi_2 = 45^o \\ & A_3 = 1.0, \quad \phi_3 = 36.0^o \\ & A_4 = 1.0, \quad \omega_3 = 14.137 \end{aligned} \tag{45}$$

In Equation (44) the first two terms are the contribution of the force of each frequency, the third term is the difference-frequency force and the last term is a noise. The computed amplitudes and phase angles are given in Table 3. The agreement between the exact and computed data is pretty good. The raw, filtered and recomputed data are plotted in Figure 9. The agreement between the filtered and recomputed data is very good.

(E) ANALYSIS OF REAL DATA

The results of the measured forces of a ship model undergoing forced roll are presented in Figures 10 through 13. Each figure consists of four different plots: (a) raw data, (b) filtered data, (c) filtered and predicted data from the method of sine function, and (d) spectrum of filtered data. The forced roll amplitude is 5 degrees and the period is 2 seconds. The axial forces (Fx) show substantial noise or the presence of other frequencies in the raw data, as shown in Figure 10. The computed amplitudes, periods, and phase angles are given in Table 4. Since the filtered data shown in Figures 10 and 11 are sinusoidal, the computed amplitudes with the three methods agree well as given in Table 4. The areas of the spectra for three forces are computed using the parabola area equation (Equation (14)) and the corresponding amplitudes are the square root of two times of the areas reported in Table 4. The axial and lateral force amplitudes are recomputed with the method of sine function and they are compared with the filtered data in Figures 10 and 11, respectively. The agreement between the filtered and predicted data is excellent.

The spectrum of the vertical force in Figure 12d shows two distinctive peaks. This indicates that the vertical force might consist of forces at two different frequencies. The spectral densities are maximal at the frequencies of 3.14 and 6.28 radians per second as shown in Figure 10. The computed amplitudes for these two frequencies are found by solving Equations (36) to (39) without including the difference terms, which are the last two terms in these equations. The amplitudes are 1.051 for the frequency of 6.28 and 0.783 for the frequency of 3.14. Their phase angles are -132.94 and –133.39 degrees, respectively. The vertical forces are recomputed with these values and they are compared with the filtered data in Figure 13. The agreement between the filtered and recomputed data is very good.

The results of the measured moments of a ship model undergoing forced roll are presented in Figures 14 to 16. The forced roll amplitude is 5 degrees and the period is 2 seconds. The lateral and vertical moments have some noise in the signal. The computed amplitudes, periods and phase angles are given in Table 5. The recomputed moments with the results of the method of sine function agree well with the filtered moments. The spectra of all moments have only one peak as shown in Figures 14d to 15d.

The results of the measured motions of a ship model undergoing forced roll are presented in Figures 17 to 19. The forced roll amplitude is 10 degrees and the period is 2 seconds. The computed amplitudes, periods and phase angles are given in Table 6. The magnitudes of both heave and pitch motions are very small compared with those of roll motions. The measured data of both heave and pitch motions are not likely sinusoidal and there appears to be an interference effect generated through the interaction between heave and roll motions or pitch and roll motions. Additional results of forced roll data analysis are included in Reference 4.

CONCLUSIONS

Methods to filter and analyze time-history data are described and a computer program HARMON is created to find the numerical results. Based on the present study the following conclusions may be drawn:

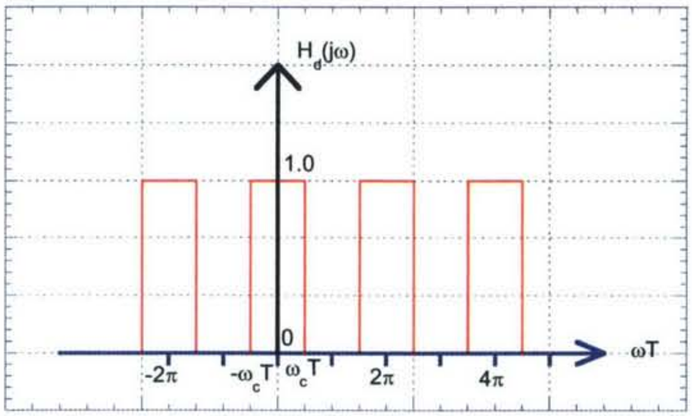
1. Since most noise of measured tank data is high frequency, the lowpass filter is a good choice to eliminate this noise. The highpass filter would be necessary to eliminate low frequency noise. The addition of the Hanning window to the filter will provide a better match to the original data.

2. When the measured data are near sinusoidal, the results of the three methods: zero crossing, sine function, and spectral analysis in computing the amplitudes, periods, and phase angles agree well as expected. When these results do not agree, it would indicate that the measured data might not be sinusoidal in nature.

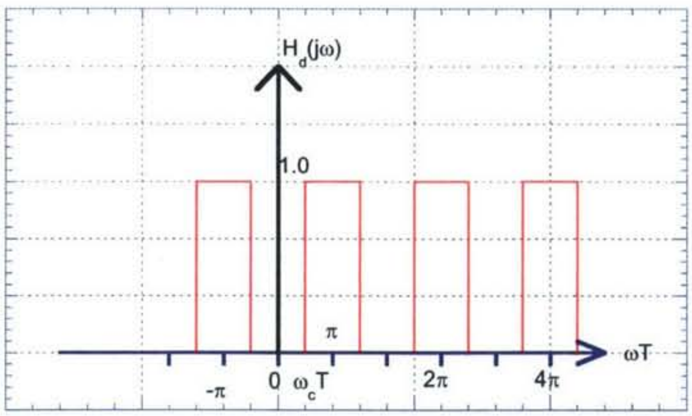
3. Each method has its own advantage. The method of zero crossing is good for the analysis of data of single frequency for the amplitude and period. The method of sine function can be applied to data of known single and two-frequency data to determine amplitude and phase angle for those frequencies. The spectral analysis method is good for data of single and multiple frequencies, and can be utilized even when the frequency of the signal is unknown.

ACKNOWLEDGMENTS

The authors would like to thank Dr. Edward Ammeen, Mr. Kurt Junghans, Dr. David Hess, and Dr. Thomas Fu for providing technical guidance.

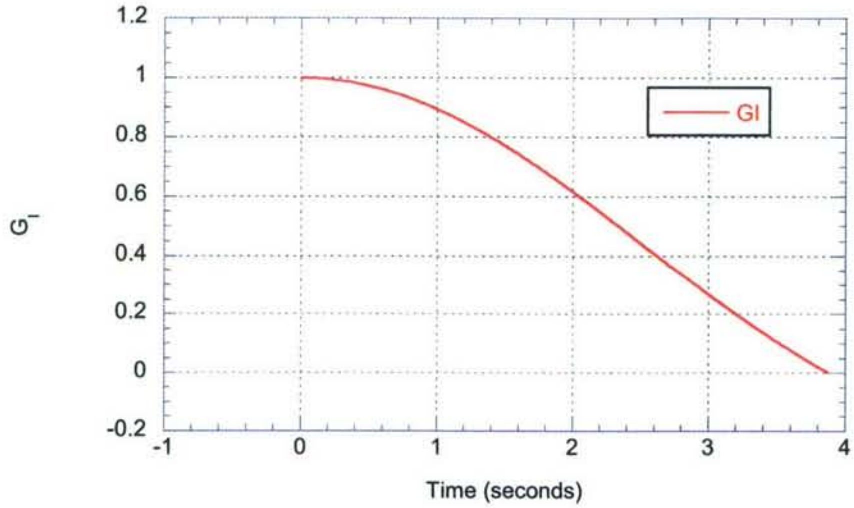


(a) Lowpass transfer function

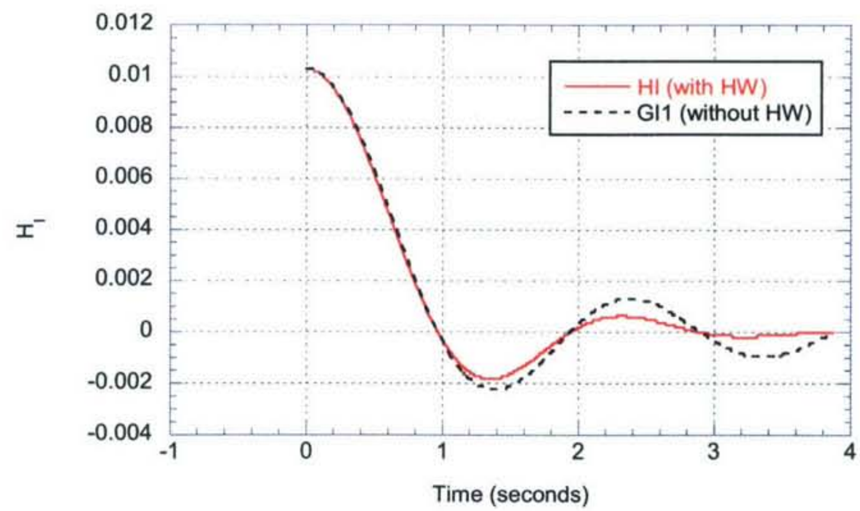


(b) Highpass transfer fuction

Fig. 1. Lowpass and highpass transfer functions

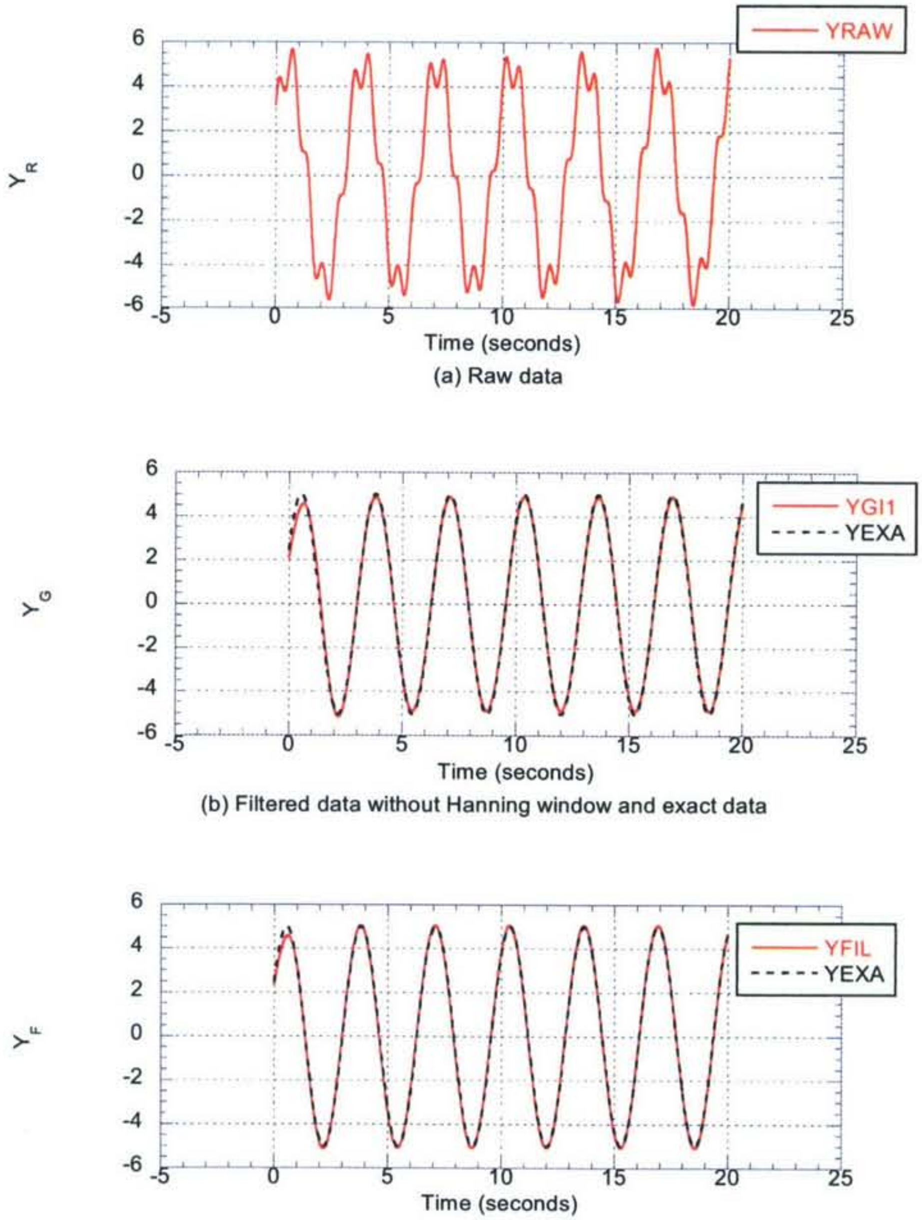


(a) Hanning window



(b) Filtering functions with and without hanning window

Fig. 2. Hanning window and lowpass filtering functions



(a) Raw data

(b) Filtered data without Hanning window and exact data

(c) Filtered data with Hanning window and exact data

Fig. 3. Raw data and filtered data with lowpass filter

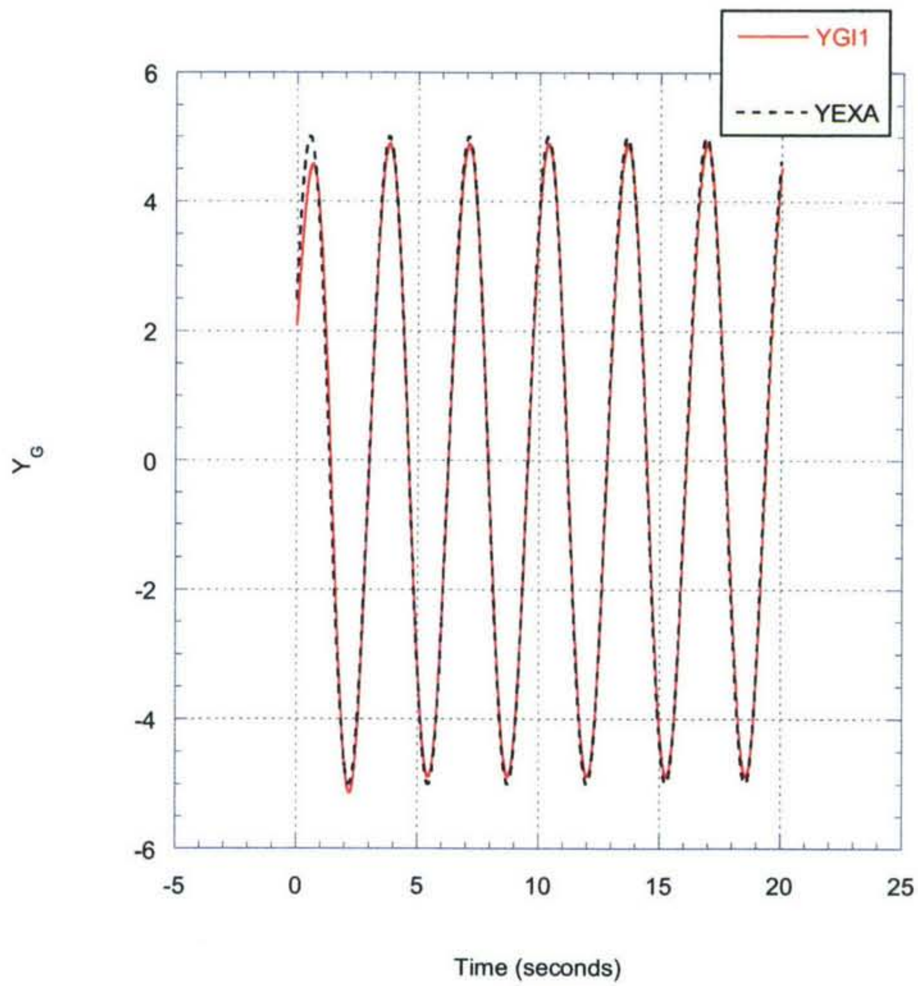
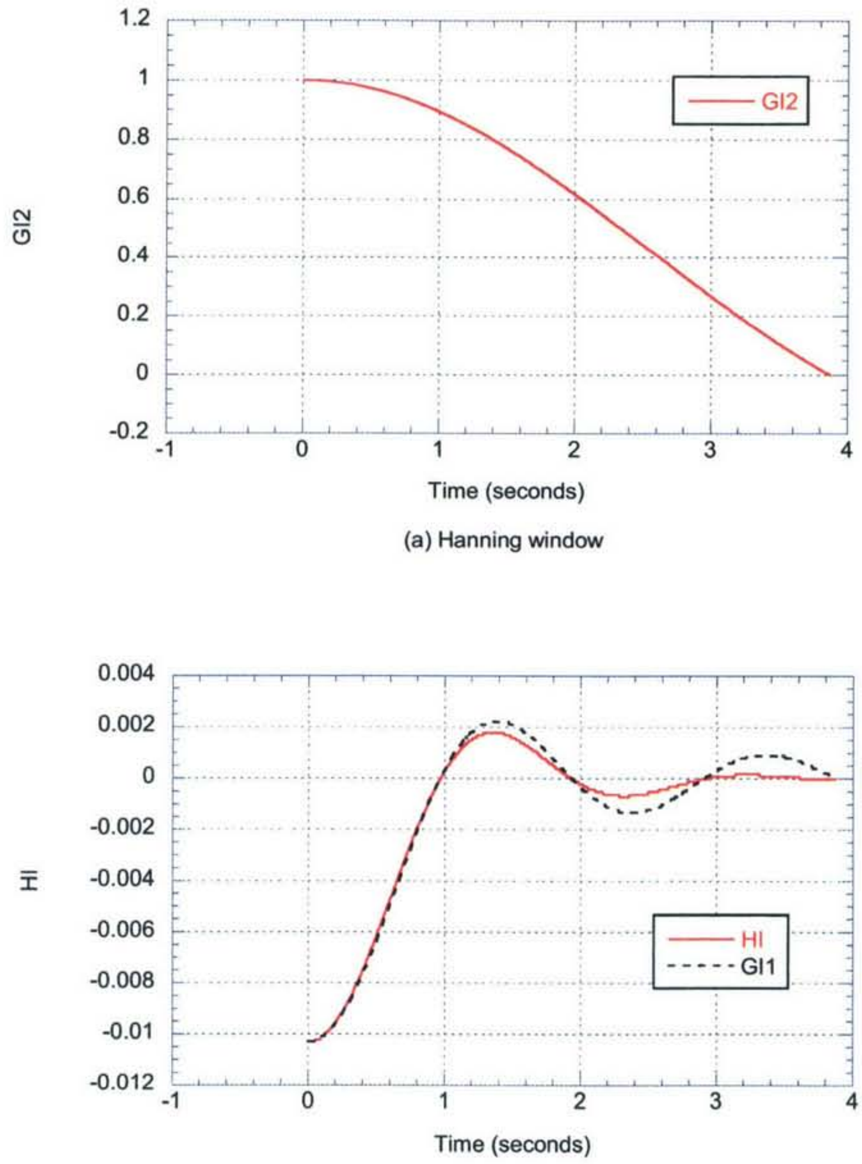


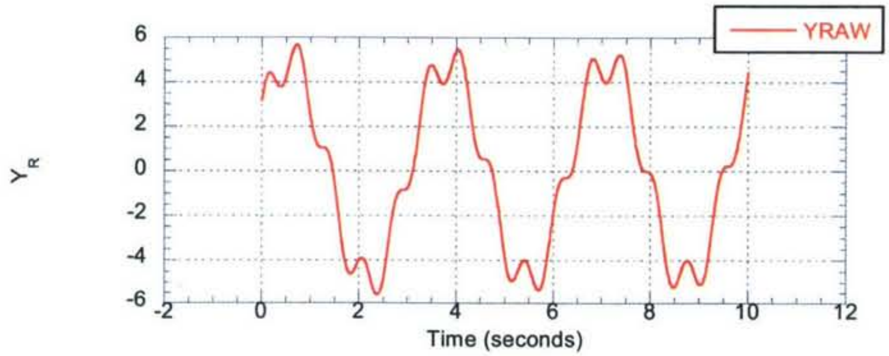
Fig. 4. Enlarged plot of filtered data without Hanning window and exact data of Figure 3b



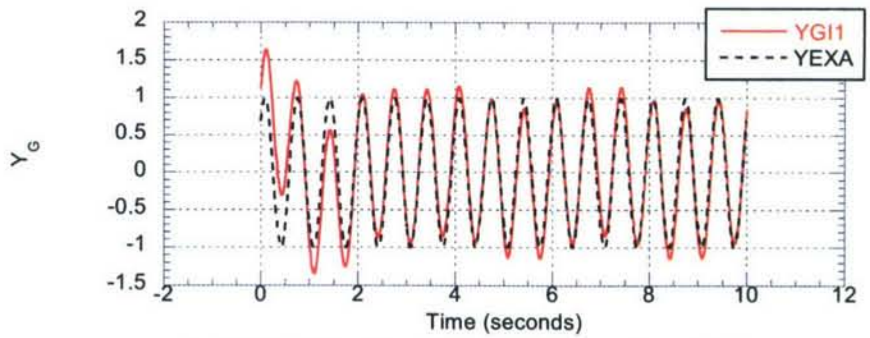
(a) Hanning window

(b) Filtering functions with and without hanning window

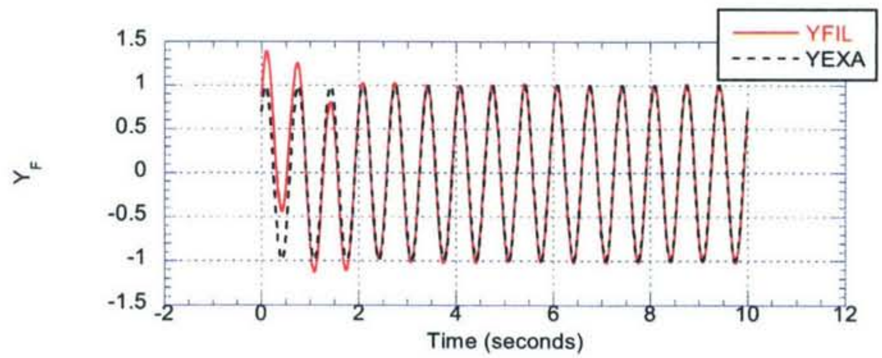
Fig. 5. Hanning window and highpass filtering functions



(a) Raw data

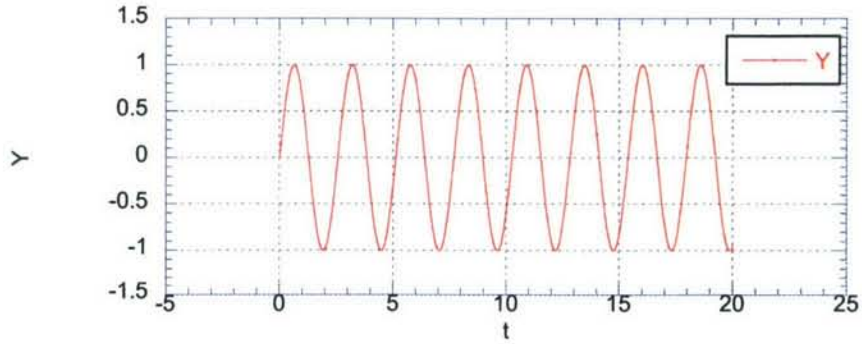


(b) Filtered data without hanning window and exact data

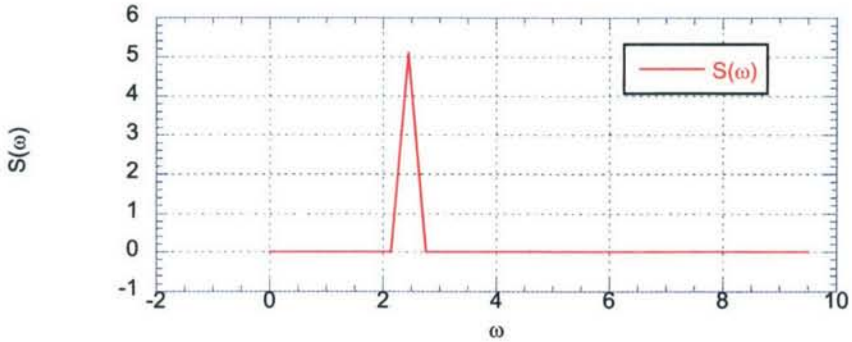


(c) Filtered data with hanning window and exact data

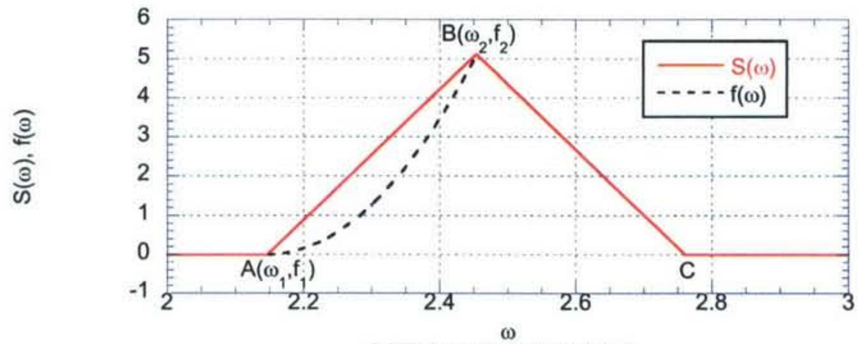
Fig. 6. Raw data and filtered data with highpass filter



(a) Sine wave of y=sin(ω_ot)



(b) Spectral density of sinω_ot



(c) Enlarged section of (a)

Fig. 7. Spectral density of a sine function

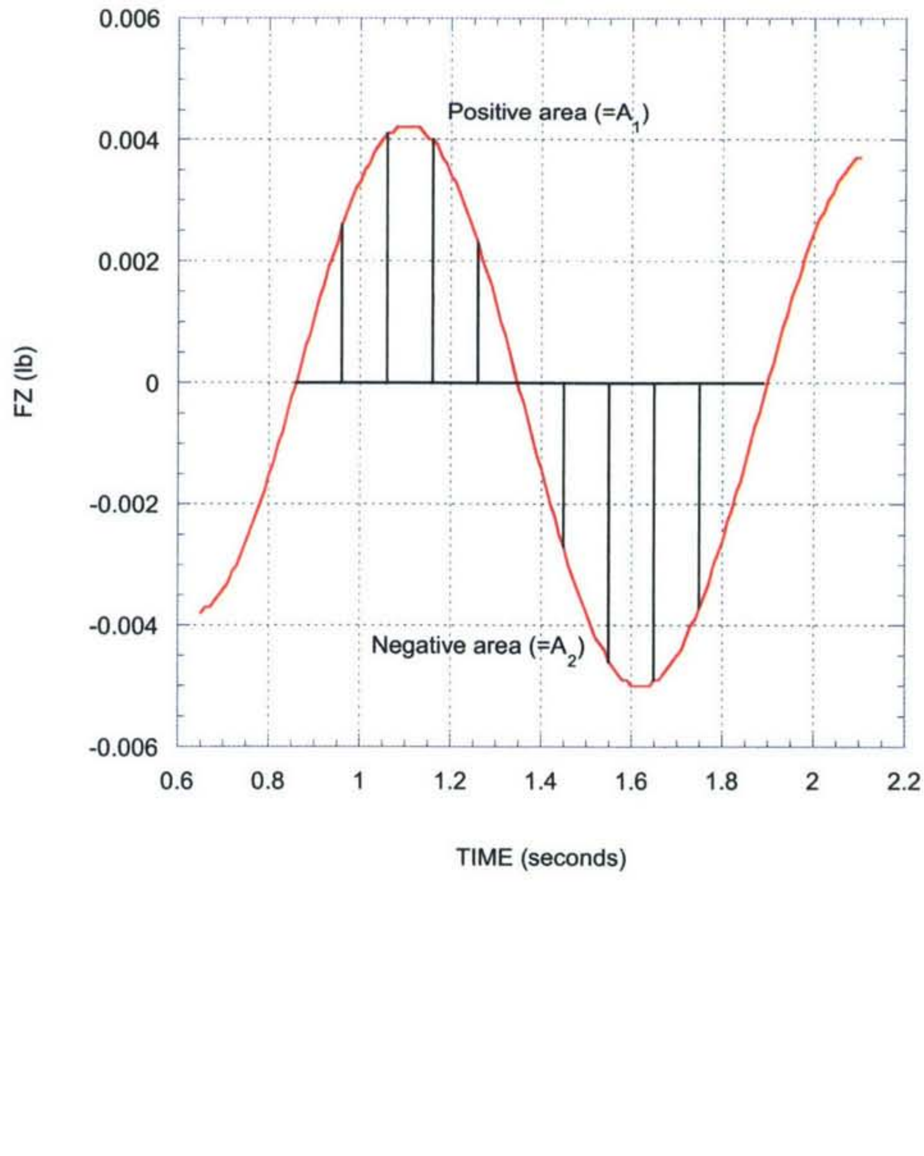
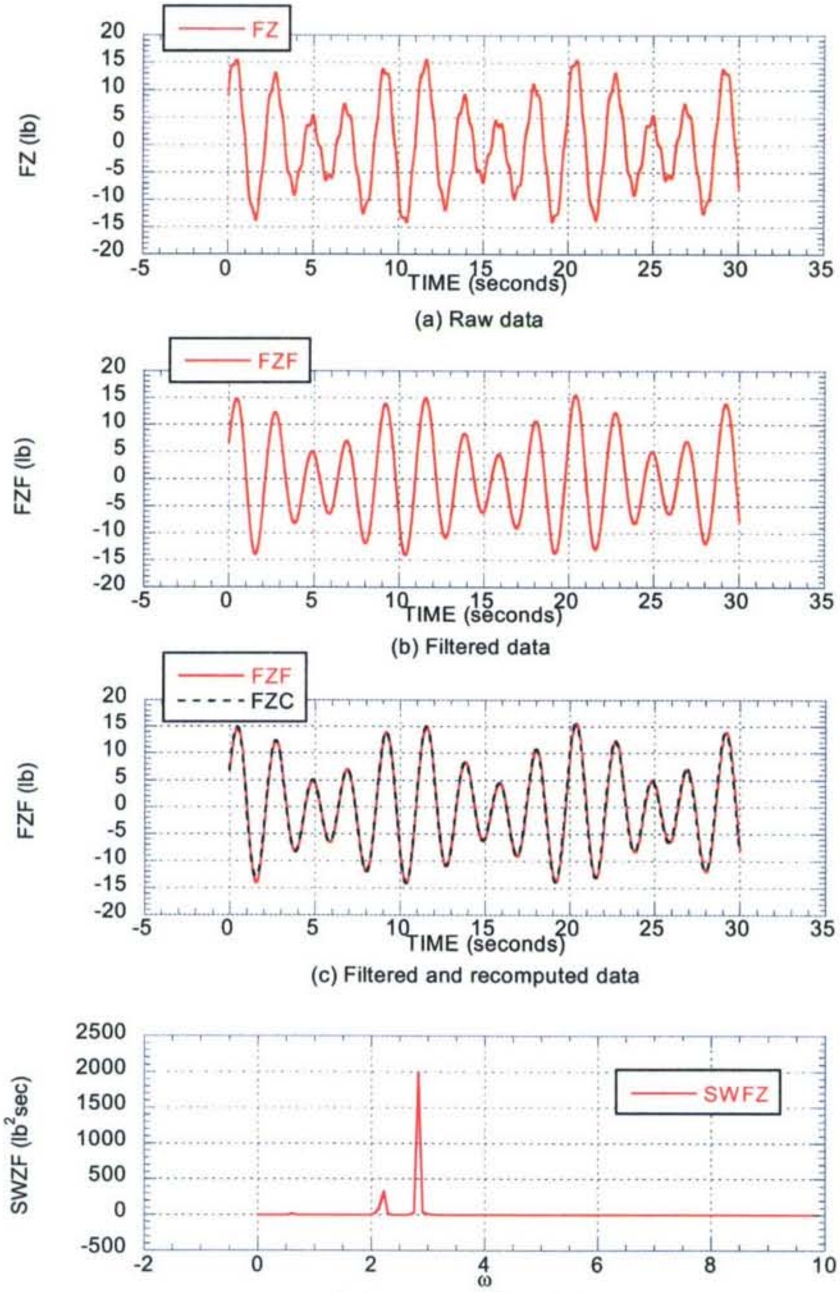


Fig. 8. Areas under half cycles of a sine wave for zero-crossing method



(a) Raw data

(b) Filtered data

(c) Filtered and recomputed data

(d) Spectrum of filtered data

Fig. 9. Vertical forces of two-frequency waves with frequencies of 2.827 and 2.199 radians per seconds

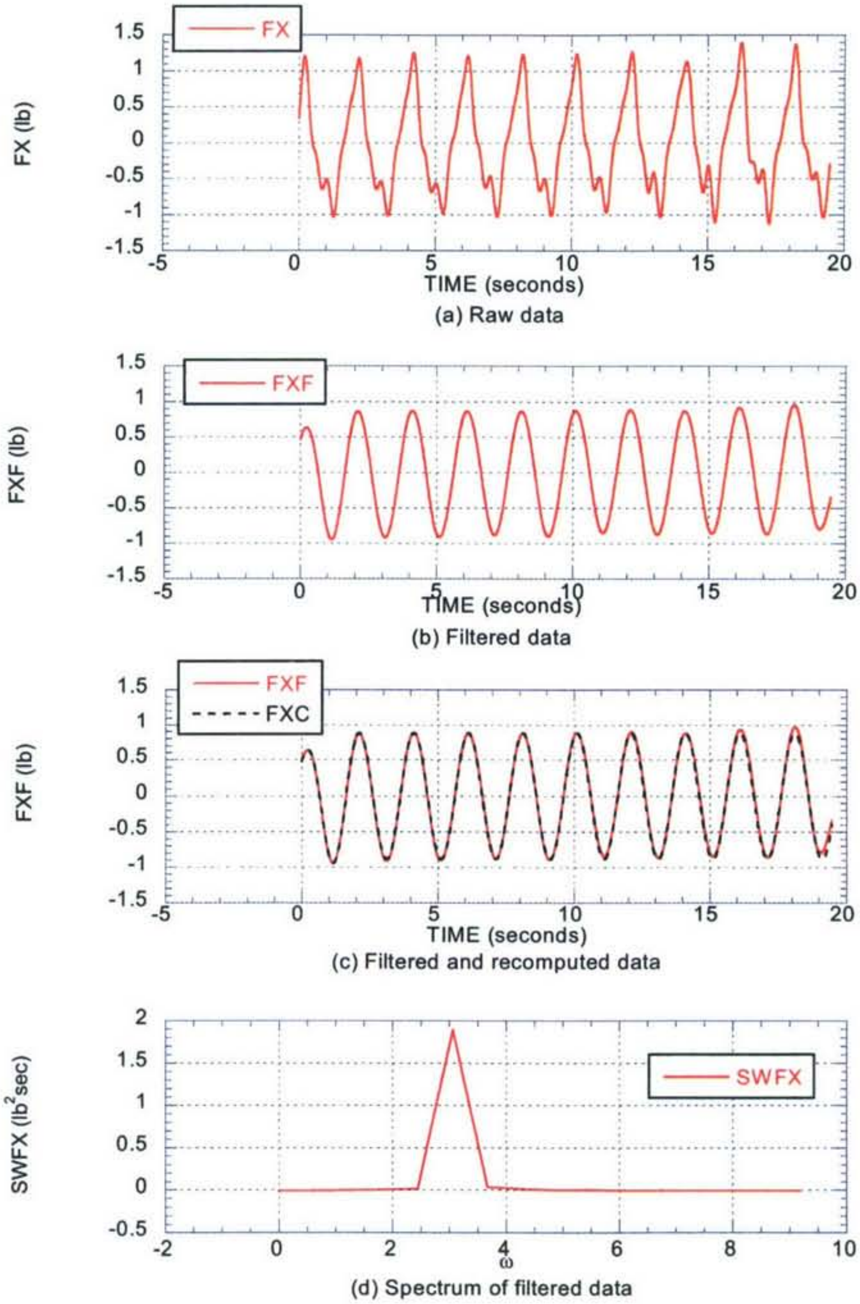


Fig. 10. Axial forces of forced roll with the roll amplitude of 5 degrees and the roll period of 2 seconds

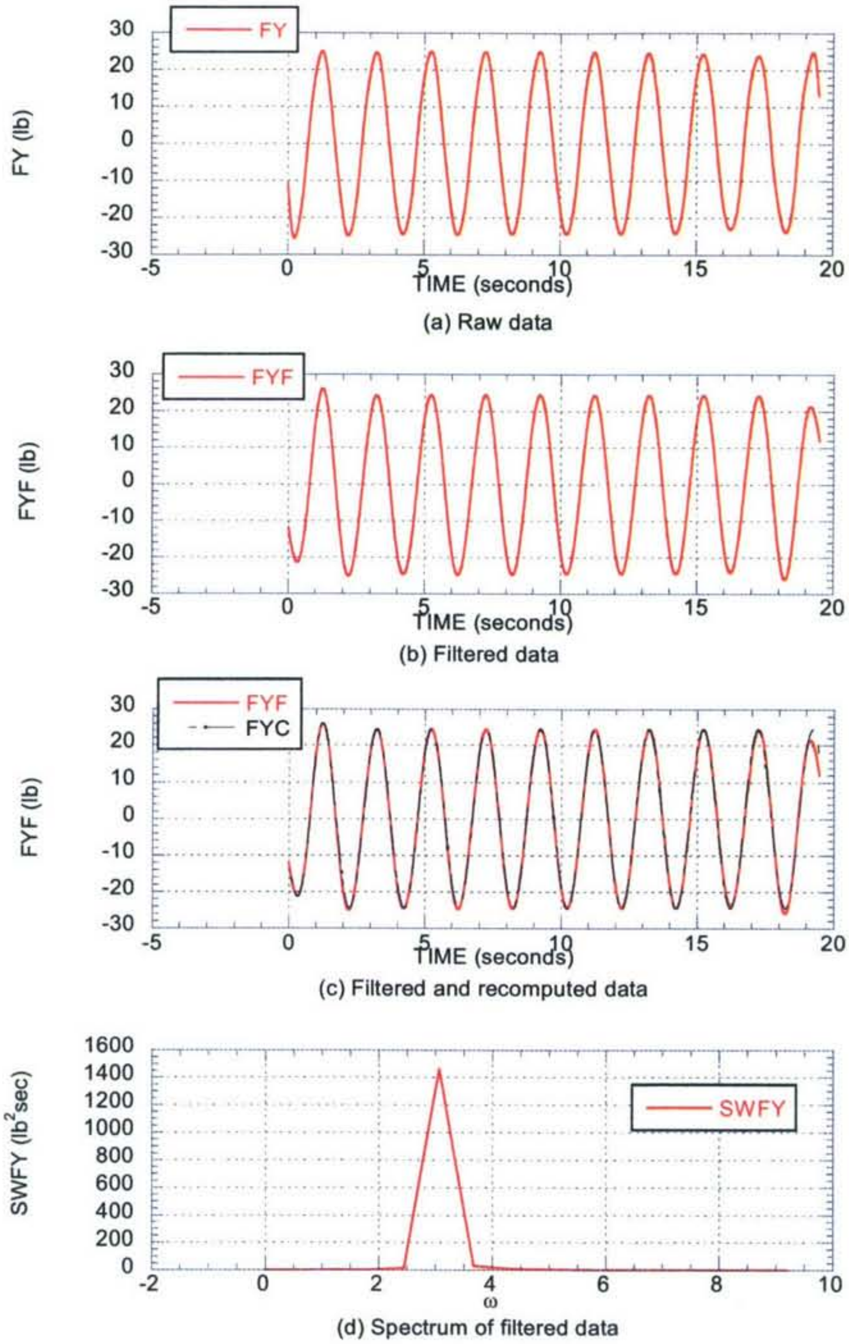


Fig. 11. Lateral forces of forced roll with the roll amplitude of 5 degrees and the roll period of 2 seconds

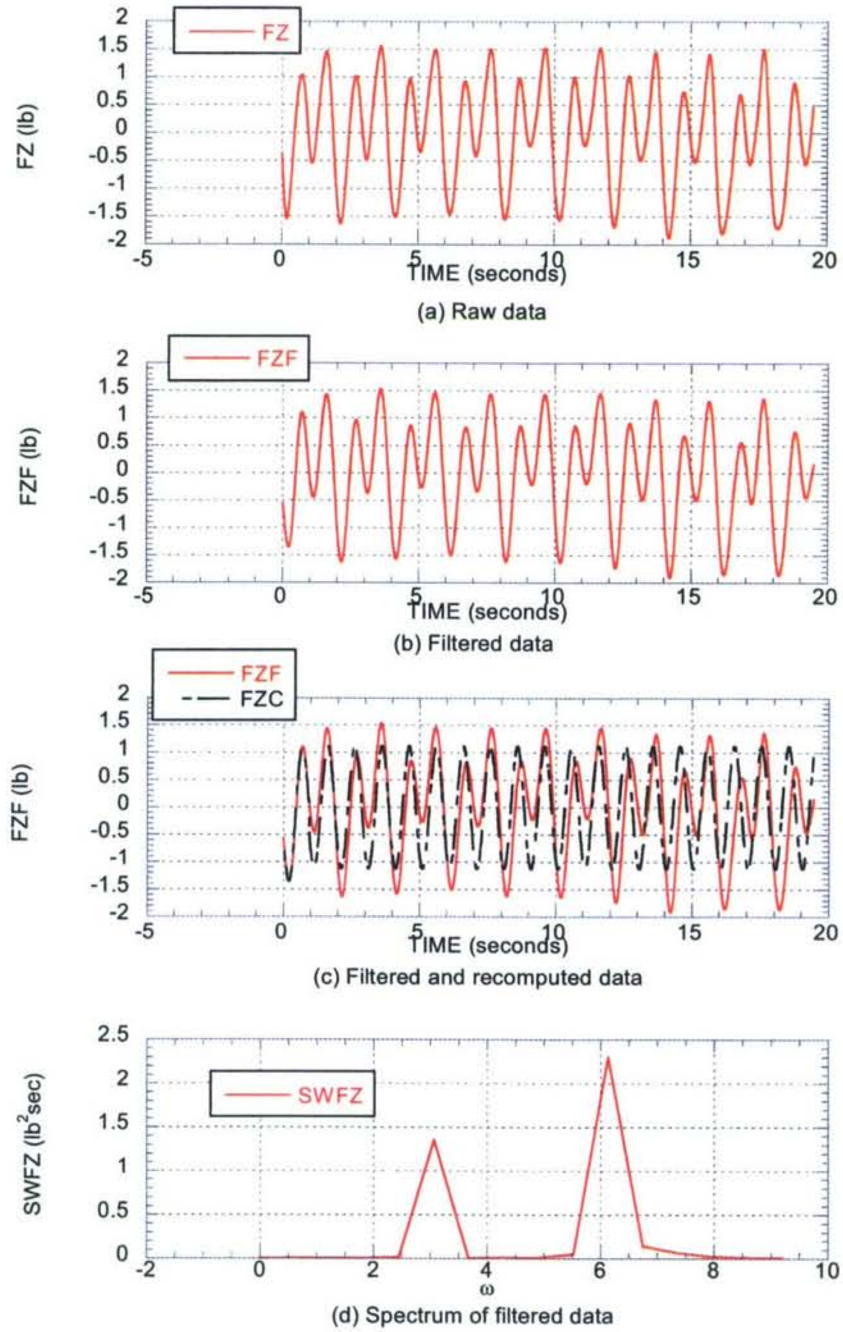
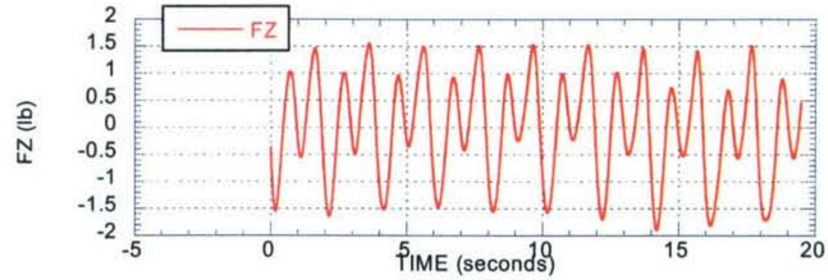
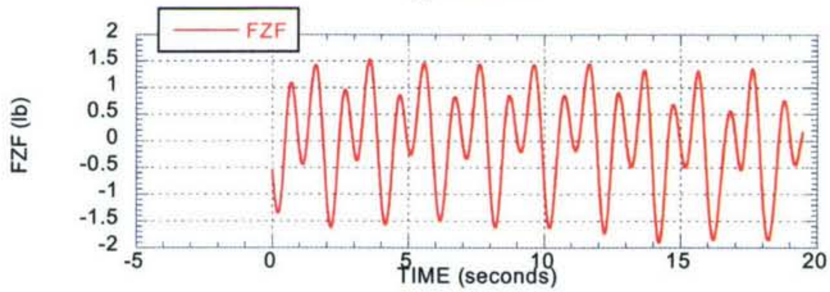


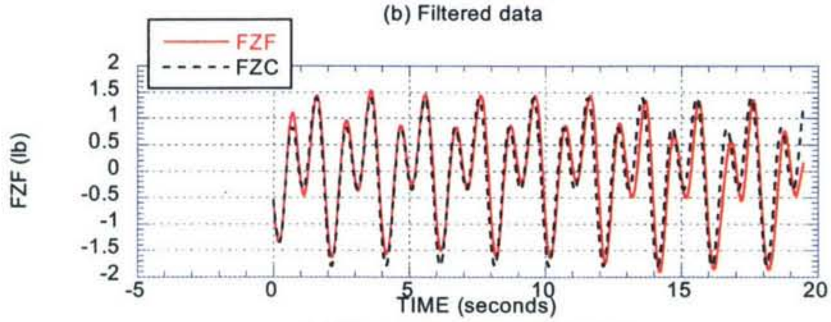
Fig. 12. Vertical forces of forced roll with the roll amplitude of 5 degrees and the roll period of 2 seconds



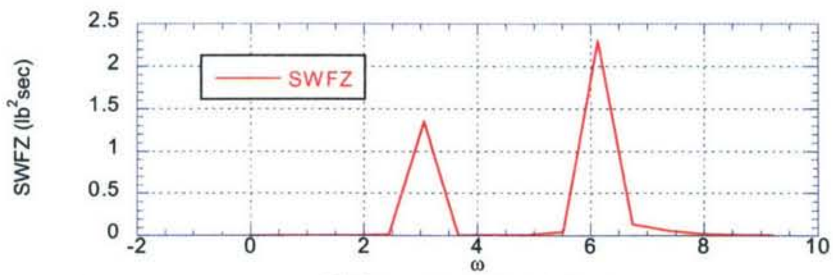
(a) Raw data



(b) Filtered data

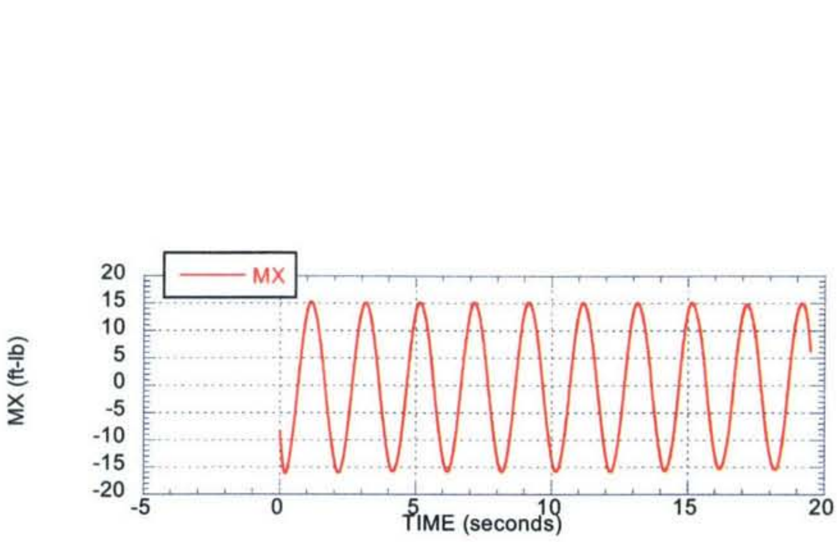


(c) Filtered and recomputed data

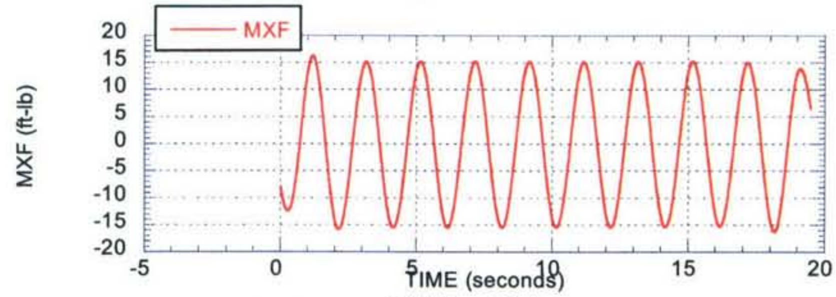


(d) Spectrum of filtered data

Fig. 13. Computation of the vertical forces with two frequencies of forced roll with the roll amplitude of 5 degrees and the roll period of 2 seconds



(a) Raw data



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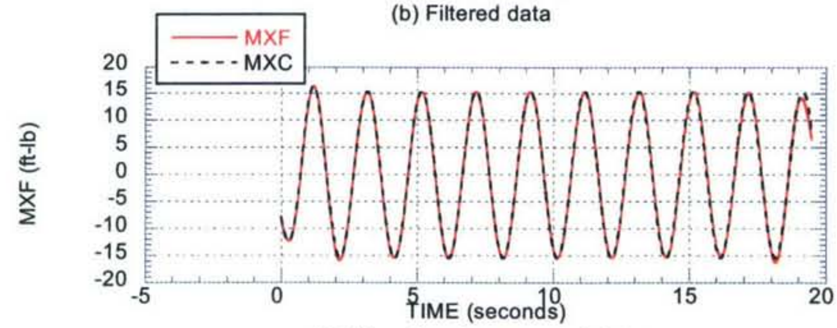
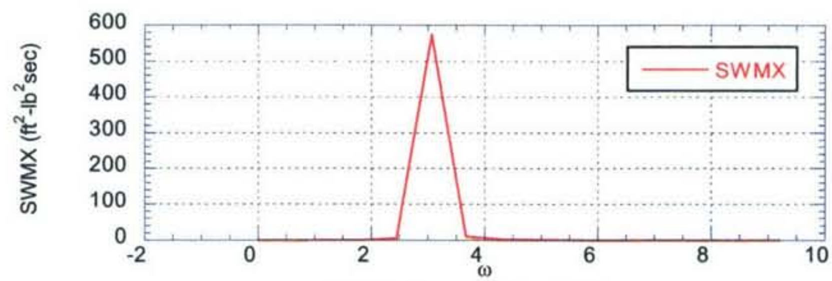
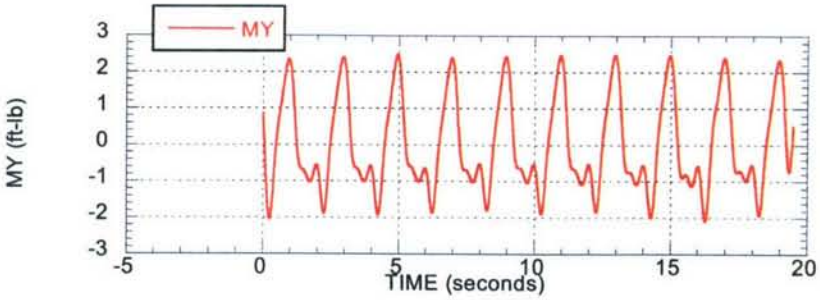


(c) Filtered and recomputed data

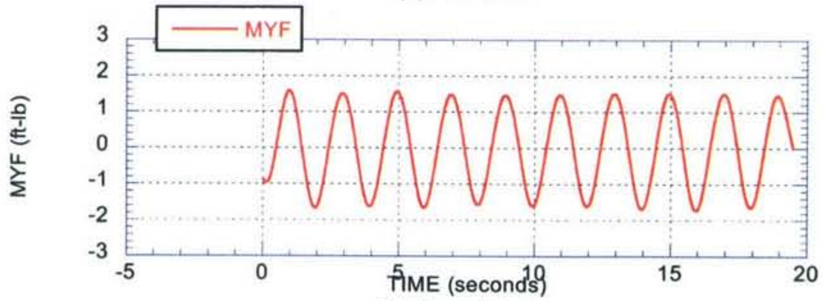


(d) Spectrum of filtered data

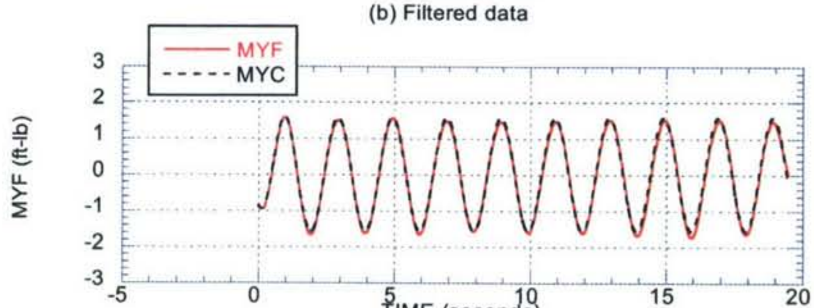
Fig. 14. Axial moments of forced roll with the roll amplitude of 5 degrees and the roll period of 2 seconds



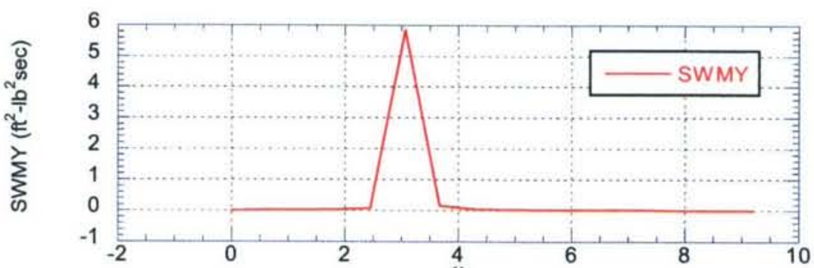
(a) Raw data



(b) Filtered data



(c) Filtered and recomputed data



(d) Spectrum of filtered data

Fig. 15. Lateral moments of forced roll with the roll amplitude of 5 degrees and the roll period of 2 seconds

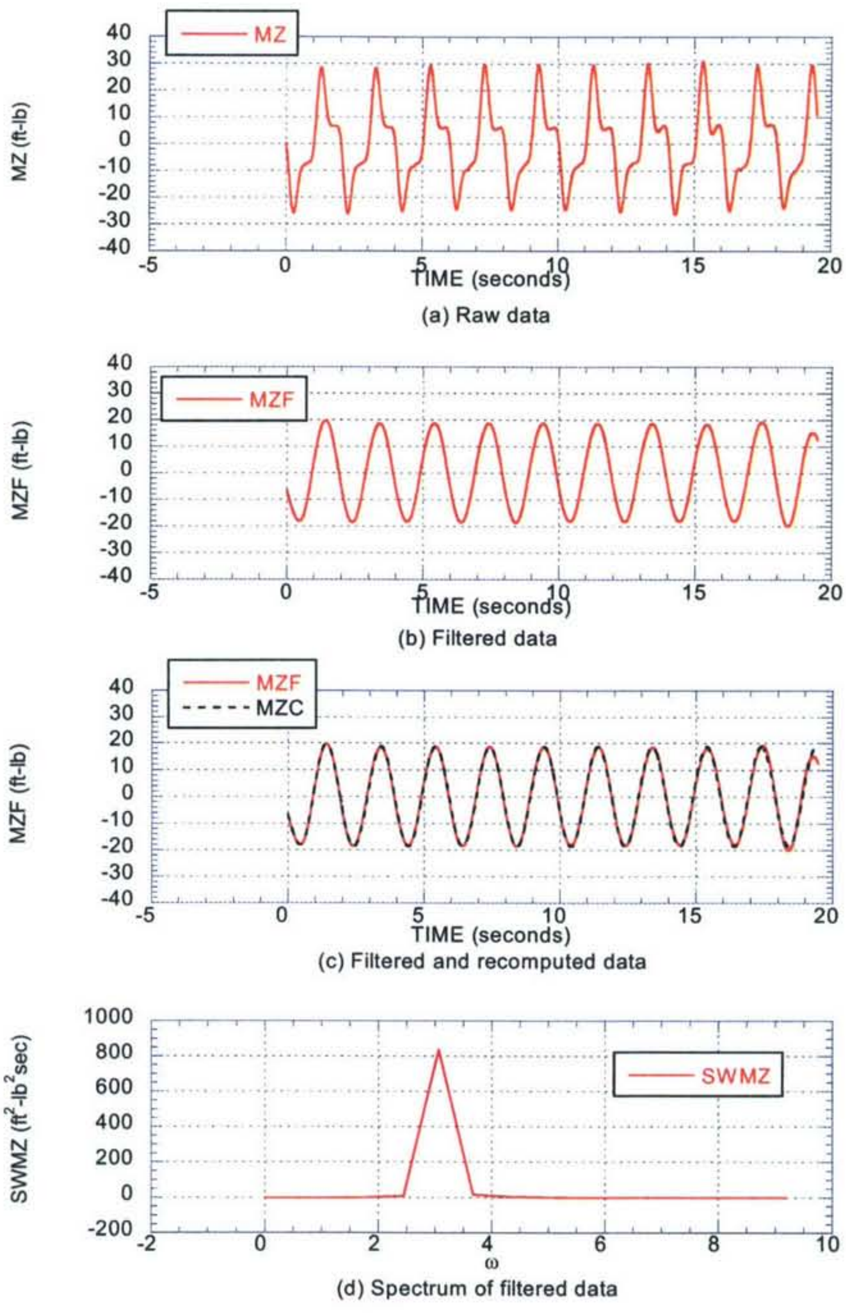


Fig. 16. Vertical moments of forced roll with the roll amplitude of 5 degrees and the roll period of 2 seconds

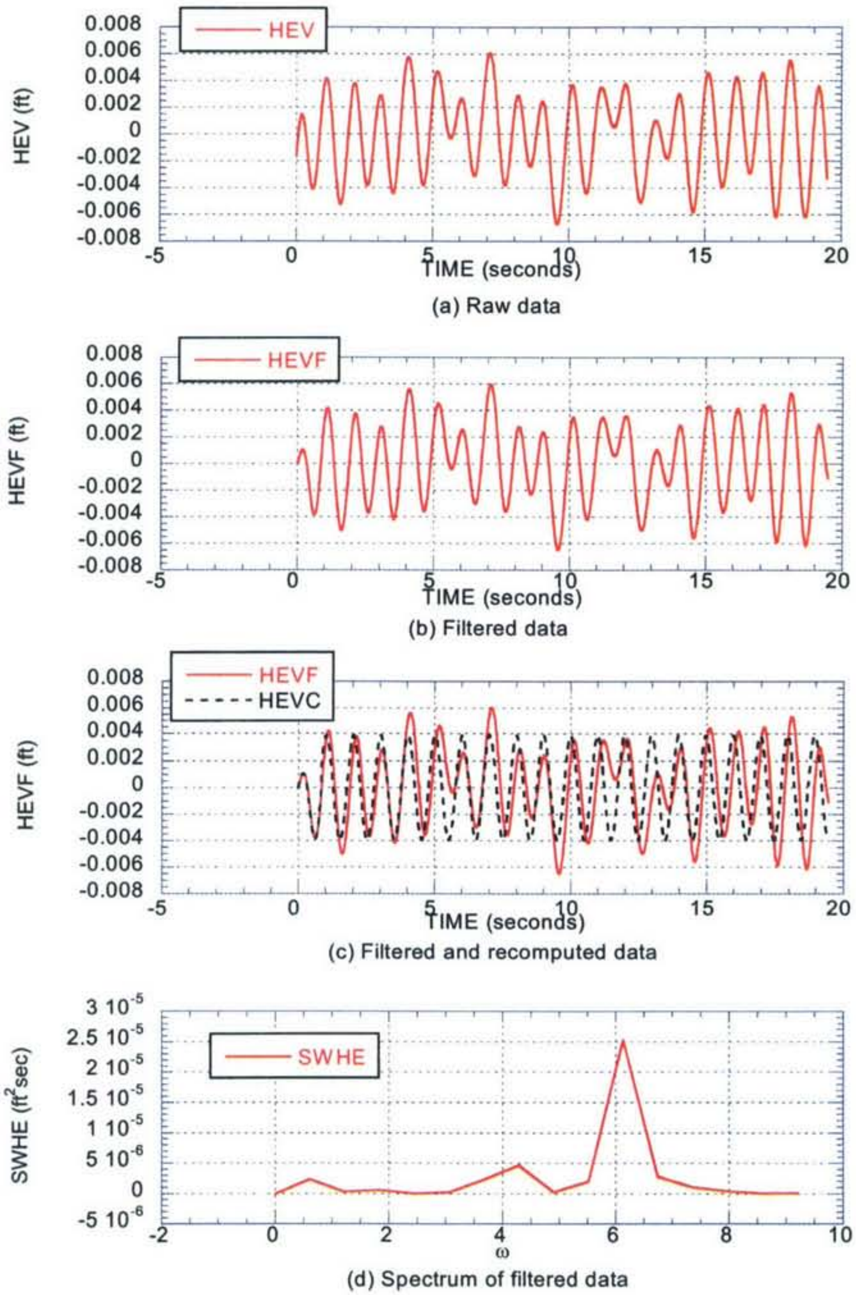
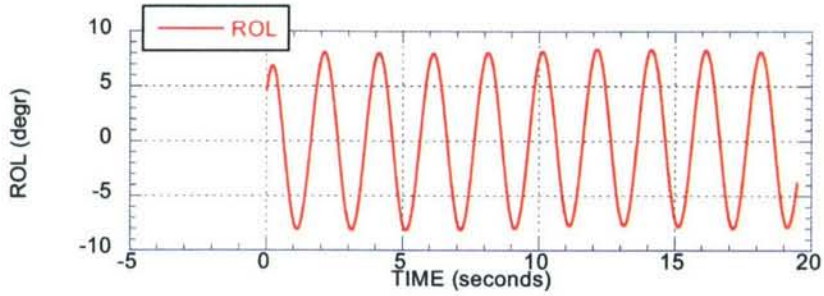
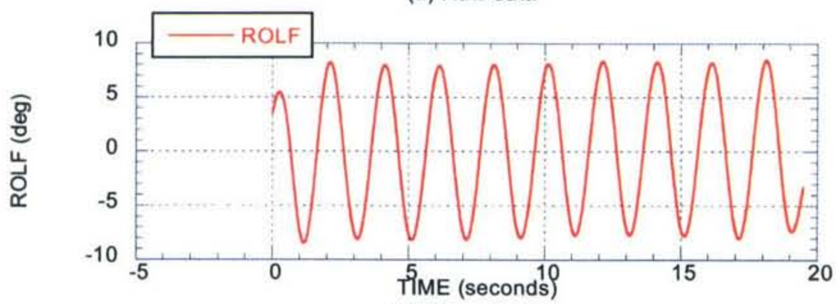


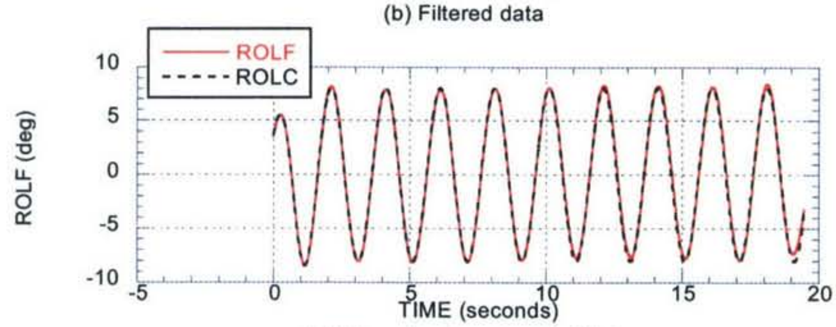
Fig. 17. Heave motions of forced roll with the roll amplitude of 10 degrees and the roll period of 2 seconds



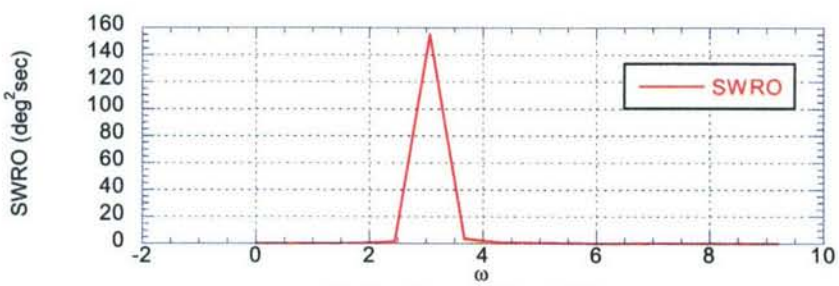
(a) Raw data



(b) Filtered data

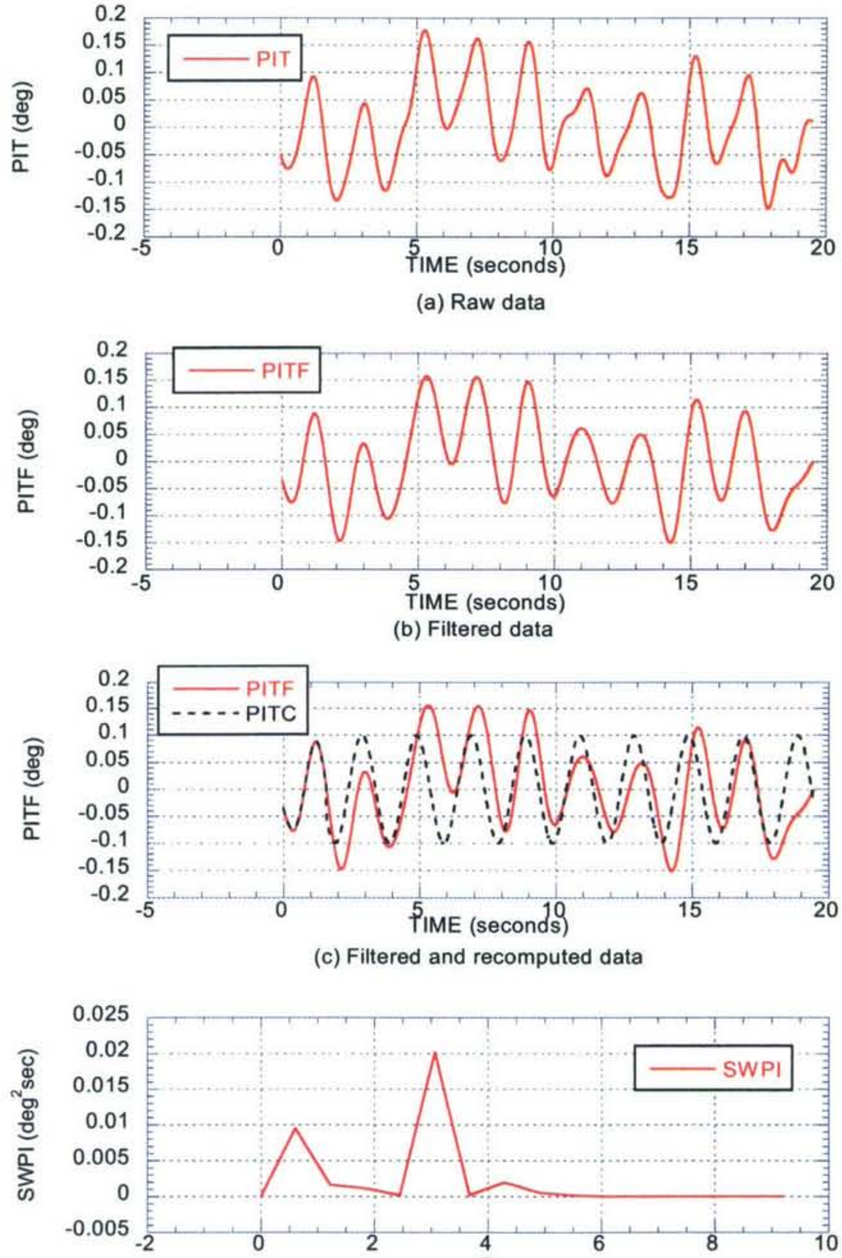


(c) Filtered and recomputed data



(d) Spectrum of filtered data

Fig. 18. Roll motions of forced roll with the roll amplitude of 10 degrees and the roll period of 2 seconds



(a) Raw data

(b) Filtered data

(c) Filtered and recomputed data

(d) Spectrum of filtered data

Fig. 19. Pitch motions of forced roll with the roll amplitude of 10 degrees and the roll period of 2 seconds

Table 1. Solution of the first term of Equation (7) with three methods

	Spectral analysis	Zero Crossing	Sine function
Amplitude (ft)	5.12	5.00	5.00
Period (sec)	3.28	3.28	3.28
Phase angle (deg)	-	-	30.00

Table 2. Solution of Equation (7) with method of sine function and spectral analysis

	Spectral analysis	Sine function
Amplitude 1 (ft)	5.12	5.00
Amplitude 2 (ft)	1.02	1.00
Period 1 (sec)	3.28	3.28
Period 2 (sec)	0.67	0.67
Phase angle 1 (deg)	-	30.00
Phase angle 2 (deg)	-	45.00

Table 3. Solution of Equation (43) with method of sine function and spectral analysis

	Spectral analysis	Sine function
Amplitude 1	10.03	10.00
Amplitude 2	5.16	5.00
Amplitude 3	1.03	1.00
Period 1 ($=T_1$, sec)	2.83	2.83
Period 2 ($=T_2$, sec)	2.20	2.20
Period 3 ($=T_1-T_2$, sec)	0.63	0.63
Phase angle 1	-	30.00
Phase angle 2	-	45.00
Phase angle 3	-	36.01

Table 4. Computed amplitudes, periods and phase angles of force data for a ship model for forced roll with roll amplitude of 5 degrees and roll period of 2 seconds

	Spectral analysis	Zero crossing	Sine function
Amplitude (lb): Fx	0.901	0.890	0.884
Fy	24.985	24.606	24.336
Fz	1.030	1.134	1.118
Period (sec): Fx	2.047	1.996	-
Fy	2.047	1.992	-
Fz	2.047	1.294	-
Phase angle (deg) : Fx	-	-	2.32
Fy	-	-	177.07
Fz	-	-	-153.57

Table 5. Computed amplitudes, periods and phase angles of moment data for a ship model for forced roll with roll amplitude of 5 degrees and roll period of 2 seconds

	Spectral analysis	Zero crossing	Sine function
Amplitude (ft-lb): Mx	15.670	15.444	15.322
My	1.587	1.572	1.566
Mz	18.921	18.686	18.671
Period (sec): Mx	2.047	1.992	-
My	2.047	1.994	-
Mz	2.047	1.994	-
Phase angle (deg) : Mx	-	-	177.70
My	-	-	-178.82
Mz	-	-	177.34

Table 6. Computed amplitudes, periods and phase angles of motion data for a ship model for forced roll with roll amplitude of 10 degrees and roll period of 2 seconds

	Spectral analysis	Zero crossing	Sine function
Amplitude: Heave (ft)	0.00355	0.00385	0.00389
Roll (deg)	8.16	8.08	8.03
Pitch (deg)	0.0920	0.0929	0.0907
Period (sec): Heave	-	1.13	-
Roll	-	1.99	-
Pitch	-	2.08	-
Phase angle (deg) : Heave	-	-	21.30
Roll	-	-	17.10
Pitch	-	-	147.00